1994

Sortie Optimization and Munitions Planning

Brown, Gerald G.

Military Operations Research

Sortie Optimization and Munitions Planning

Gerald G. Brown
Operations Research
Naval Postgraduate School
Monterey, CA 93943

Dennis M. Coulter
Headquarters, USAF
Pentagon
Washington, DC 20330

Alan R. Washburn
Operations Research
Naval Postgraduate School
Monterey, CA 93943

ABSTRACT

Since 1965, the United States Air Force has relied on mathematical programming for the planning of conventional air-to-ground munitions. The centerpiece of this planning effort is HEAVY ATTACK, a theater-level model employing large-scale nonlinear programming to load weapons onto aircraft and assign sorties to targets. The single-period objective is to maximize the expected destroyed target value over the forecast weather states by assigning sorties which use the best delivery tactics in each weather state with available aircraft and weapons stocks. Over multiple periods, HEAVY ATTACK accounts for differences between targets in regeneration rate, value, and ease of damage assessment, and evaluates aircraft attrition and remaining weapons stocks, mounting the best sorties possible with the remaining resources. In 1988 approximately $2 billion worth of weapons were purchased with guidance from HEAVY ATTACK; additional expenditures of $5.2 billion are being planned for 1994—99.

In 1990—91, media coverage of Desert Storm made the focus of HEAVY ATTACK apparent to millions of viewers.

> As many arrows, loosed several ways, _ so may a thousand actions, once afoot, end in one purpose, and be all well borne without defeat.
>
> *Shakespeare (Henry V)*

INTRODUCTION

The United States Air Force (USAF) bases its air-to-ground munitions planning on the projected need for weapons in fighting a protracted war. Sufficient stocks of such weapons must be in place in, or transportable to, a theater of operations for timely use in combat. In order to determine the required stores of such weapons, some evaluation of their use in hypothesized theater-level conflict is required.

Over the past 25 years, the USAF has pioneered in the modeling and optimization of the end effects of the procurement, stockpiling, and combat use of conventional air-to-ground munitions; the goal is to provide guidance credible to military planners, to the Legislative and Executive branches of the U.S. Government, and, ultimately, to U.S. taxpayers. USAF is unique amongst the military services in the extent to which it relies on mathematical programming to accomplish this. In 1988 approximately $2 billion worth of weapons were purchased, and expenditures of $5.2 billion are already planned for 1994—99 (e.g., Department of Defense [1993]) with guidance from HEAVY ATTACK, the main subject of this paper. HEAVY ATTACK is one of the major applications of mathematical programming in the United States. In this article we review the history of the system, describe its current use, and project near-term enhancements based on current research.

BACKGROUND

The USAF is interested in optimization because aircraft are flexible weapon systems; depending on how an aircraft is loaded with weapons, it can more or less efficiently attack a variety of targets. Given a collection of several types of aircraft (say a_i aircraft sorties of type i; $i = 1,\ldots,A$) to be used in attacking a collection of targets (say t_j targets of type j, $j = 1,\ldots,T$), the problem of assigning aircraft to targets naturally arises. Perhaps the simplest formulation of the problem would be to let E_{ij} be the average number of targets of type j killed by a sortie of type i, x_{ij} the number of sorties of type i assigned to targets of type j, and then solve program **LP1:**

(LP1)

$$\max_{x,y} \sum_{j=1}^{T} v_j y_j$$

$$\text{s.t.} \quad \sum_{i=1}^{A} E_{ij} x_{ij} = y_j;\ j=1,\ldots,T \tag{1}$$

$$\sum_{j=1}^{T} x_{ij} \le a_i;\ i=1,\ldots,A$$

$$x_{ij} \le 0,$$

where y_j (a variable) is the average number of targets of type j killed and v_j (an input) is the subjectively-assessed value of a target of type j. The meaning of the objective function is "average value of targets killed".

Although (**LP1**) is a good starting point for this exposition, USAF has not to our knowledge ever actually used it. The difficulty is that solutions to such formulations tend to be very extreme in nature. Each aircraft type is entirely assigned to a single target type (i is assigned to j if j maximizes v_jE_{ij}); it is even conceivable that all aircraft types might be assigned to the same target type. Although this kind of solution might be reasonable in a target-rich environment where sorties are hopelessly outnumbered by targets, it is neither realistic nor acceptable in general, even merely for planning purposes.

There are two direct methods of embellishing **(LP1)** so that the solutions are not so extreme. The simpler is to add the constraints $y_j \leq t_j$ (note that **(LP1)** does not involve the data t_j at all) to prevent the possibility of killing more targets on average than are known to exist. Call the resulting linear program **(LP2)**. In **(LP2)**, sorties may be assigned to targets other than their favorite type if the favorite type is exhausted. The Theater Attack Model (TAM) discussed by Might [1987] is a linear generalization of **(LP2)** where variables have additional subscripts corresponding to weather, weapons, etc., so descendants of **(LP2)** are well-represented amongst contemporary military planning models. However, even though **(LP2)** accomplishes the goal of cutting off the extreme solutions while introducing minimal complication to **(LP1)**, it was not the method originally chosen by USAF.

The other direct method of fixing **(LP1)** is to make the objective function *nonlinear* to reflect the idea of decreasing returns as y_j is increased. An early USAF model, SABER MIX, was identical to **(LP1)** except that the objective function was

$$\sum_{j=1}^{T} v_i t_j (1-\exp(-y_j/t_j)) \qquad (2)$$

This objective function might be justified by arguing that the "fog of war" will cause the statistics of the number of times a particular target of type j is killed (say, X_j) to obey the Poisson distribution (Blackett [1962]). Since the expected value of X_j is y_j/t_j, it follows from the Poisson assumption that the probability that any target of type j is *not* killed is

$$P(X_j=0)=\exp(-y_j/t_j).$$

Equation (2) then follows directly. The objective function still has the meaning "average value of targets killed", just as in **(LP1)** and **(LP2)**. Of course the new mathematical program (call it **(NLP1)**) is of a more difficult type; the constraints are linear, but the objective function is not.

Since $1 - \exp(-x) \leq x$ for $x \geq 0$, **(NLP1)** will always have a smaller optimized objective function than **(LP2)**. **(LP2)** essentially incorporates the assumption that sorties can be coordinated so that targets that have already been killed will not be further attacked. This kind of coordination is assumed to be impossible in **(NLP1)**, with the attendant possibility of wastage due to overkill. The two programs correspond to extreme assumptions about the kind of command and control that can be exercised in battle.

(NLP1) was a nontrivial optimization problem when it was formulated in the 1960's. An early attempt at a solution involved the assumption that *all* targets were to be attacked by *one* aircraft type. This problem has the same mathematical form as the corresponding Search Theory problem of allocating random search effort to a collection of cells (Charnes and Cooper [1958]), so an efficient solution technique was available by the time the Munitions Planning Branch was formed. However, there were obvious problems with assigning each aircraft type as if none of the others existed, so the desirability of solving the joint optimization problem where all aircraft types are considered simultaneously was quickly recognized.

In the early 1970's, the Directorate of Defense Program Analysis and Evaluation (DDPA&E) funded a mathematical programming-based scheme for assigning aircraft to targets that incorporated the best features of SABER MIX and two other systems that were then in use. The resulting formulation **(NLP2)** is reported in Clasen, Graves, and Lu [1974]. The **(NLP2)** objective function is similar to that of SABER MIX except for the incorporation of an additional parameter c_j:

$$\sum_{j=1}^{T} v_j t_j (1-\exp(-c_j y_j/t_j))/c_j. \quad (3)$$

Note that (3) is the same as (2) if $c_j = 1$, and that (3) is the same as (1) in the limit as c_j approaches 0. The parameter c_j thus bridges the situation where command and control is impossible (the Poisson case $c_j = 1$) and where it is perfect (the linear case where c_j approaches 0). However, a precise meaning for c_j has never been given. Clasen, Graves, and Lu say only that "DDPA&E suggested this function to us. It is similar to the objective function of the SABER MIX methodology." The lack of a physical meaning has proven to be troublesome for subsequent generations of USAF officers (see Embellishments) required to estimate c_j for various classes of targets, and various levels of engagement "fog" in the theater.

(NLP2) also incorporates constraints on y_j that ensure that not more than t_j targets of type j are killed, on average. Thus **(LP2)** and **(NLP1)** are both special cases.

HEAVY ATTACK

The HEAVY ATTACK model currently in use by the USAF Weapons Division is still, at its heart, the Clasen, Graves, and Lu model **(NLP2)**, but now larger and embellished with additional types of linear constraints. HEAVY ATTACK considers a sequence of allocation problems, with each problem corresponding to one time period in a war that is projected to be several time periods long. Targets that survive the attacks of one period are still available in the next, together with reinforcements and also with targets that were killed in previous periods but have since been repaired (reconstituted), possibly with different values. The optimization is done myopically, with the objective in each period being to kill as much target value as possible without regard to the effect on future periods. The myopic feature is analytically convenient, since it permits the analysis of a sequence of small problems rather than one large one, but it is also realistic in the sense that the actual policy for assigning aircraft to targets (which must be distinguished from the value-based method in HEAVY ATTACK) is a joint-service process that does not include the idea of "saving targets for the future". There may be an element of making virtue out of what was once a necessity here, but still that is the justification usually given for myopia.

HEAVY ATTACK depends on a separate program, SELECTOR, for the sortie effectiveness inputs E_{ij}. SELECTOR is needed not because effectiveness coefficients would otherwise be lacking, but rather because there are too many of them. The Joint Munitions Effectiveness Manual (JMEM, e.g. Joint Technical Coordinating Group/Munitions Effectiveness [1980]) shows how to tabulate

E_{ijtw} = average number of targets of type j killed by a sortie of type i using tactic t in weather type w.

SELECTOR's role is essentially to get rid of the last two subscripts. The method for doing this is important, since it is often the case that the most effective tactics are associated with expensive munitions or high attrition to the delivering aircraft. SELECTOR adopts only the most cost-effective tactic: literally the tactic that maximizes the ratio of sortie cost (including the cost of weapons used and expected attrition) to E_{ijtw}. Let this tactic be $t^*(i, j, w)$, and let $E_{ij\cdot w}$, be the effectiveness when that tactic is used. The coefficients E_{ij} required by HEAVY ATTACK for *each period* are obtained by simply averaging $E_{ij\cdot w}$ over whatever weather distribution is appropriate in the area of the supposed conflict; the natural notation would be $E_{ij\cdot\cdot}$, but we will drop the two dots to be consistent with earlier usage. $t^*(i, j, w)$ may change from period to period as stocks of the requisite weapons become exhausted. The weather distribution may also change from period to period, so the same is true of E_{ij}. Crawford [1989] concludes that usage of HEAVY ATTACK with E_{ij} computed in this manner biases weapons purchases towards cheap but inefficient weapons. He reasons that the real "cost" of attrition considerably exceeds the aircraft flyaway cost used by USAF in determining "bang-per-buck."

The inputs to HEAVY ATTACK are determined once each year for each potential theater of operations. The Weapons Division hosts an annual theater planning session attended by mid-level operational, intelligence, logistics, and planning staff officers. The goal is to produce a realistic current requirement for theater weapons stocks. Attendees must travel great distances to participate in these sessions, and can only be expected to remain briefly in residence before returning to exigent duty. Cadre analysts (e.g., Coulter) are responsible for care and feeding of HEAVY ATTACK, and thus must interpret the proposals of the theater planning group and endeavor to provide compelling scenarios for their evaluation. Once HEAVY ATTACK has determined sortie-to-target allocations for each period of the war, munition requirements can be recovered by recalling the function $t^*(i, j, w)$ determined by SELECTOR and doing the appropriate accounting. The resulting estimates of theater weapons stocks requirements are used to justify the aggregate USAF annual weapons buy request. These budget requests are exposed to exhaustive scrutiny by USAF, and subsequently by other levels of review. Budget revisions by higher authority are reconciled with mission requirements with the help of HEAVY ATTACK.

EMBELLISHMENTS

HEAVY ATTACK as first formulated by Clasen, Graves, and Lu in 1974 was a nonlinear program with at most 10 sortie types and 45 target types; even with their new method, solving problems of that size required quite a while on contemporary computers. Enriching the model in any manner that would have increased run times was out of the question, however nervous its users might have been about such things as the SELECTOR/HEAVY ATTACK system's myopic approach to optimization over time, fractional sortie assignments, and the suboptimization implicit in SELECTOR's preprocessing of the JMEM effectiveness data.

However, computers and computer software have improved substantially since 1974. Lord [1982] reports mainframe solution times measured in seconds, rather than minutes, upon completing the installation of the X-system solver (e.g., INSIGHT [1990], and Brown and Olson [1994]). Bausch and Brown [1988] describe a prototypic implementation of HEAVY ATTACK on an 80386-based IBM-compatible microcomputer, an uncommon feat at the time. In 1991 more powerful 80486 machines were configured for production use in various environments, implemented with mainframe-compatible software (Silicon Valley Software [1991]), and shipped to users as HEAVY ATTACK machines. Wallace [1992] exploits this new capability by designing and implementing a prototypic graphical user interface (GUI) which is especially useful for comparing outputs from multiple scenarios. Bradley, et al. [1992] give an unclassified demonstration of this unified hardware and software decision support system: sortie optimization for 25 aircraft types, 90 weapon types, and 100 target types, problems with hundreds of constraints and thousands of variables for each of 6 time periods, requires about two minutes from SELECTOR input to final output. Washburn [1989] describes a new method tailored to the HEAVY ATTACK problem that solves (**NLP2**) with 13 sortie types and 61 target types in about two seconds on an 80386 machine. Plainly, the computation time considerations that drove the original (**NLP2**) formulation have now been substantially relaxed, to the point where more computationally stressful reformulations can be considered.

However, SELECTOR/HEAVY ATTACK's long and successful lifetime as a planning tool makes it difficult to consider *any* substantial reformulation, even now that computational cost is negligible. Generations of Air Force officers have learned to cope with the idiosyncrasies, assumptions, and data requirements. Inter-organizational relationships have evolved to provide inputs and interpret outputs. *"If it ain't broke don't fix it."*

So the Weapons Division is naturally attracted most by changes that merely affect computational efficiency or ease of use. For example, linear side constraints can now be used to insure flyable mixtures of sorties. Graphical user interfaces are expensive to design and develop, but are invaluable for quick, reliable formulation of input scenarios and interpretation of their output. Further, although HEAVY ATTACK can run on many hardware platforms, and employs the fastest large-scale optimizer in our experience, a 486 PC is now the favored host due to its low cost, convenience, and portability. In early 1993, theater conferences were completed for the first time without requiring the physical attendance of the theater planners.

Less favored, but sometimes essential, are changes that produce the same output quantities from the same input quantities. For example, a variety of changes have been made to the target reconstitution model. These changes do not require new inputs nor change the meaning of the outputs, although the output quantities are of course affected.

Least favored are reformulations that require an essentially different way of looking at things. Recently, constraints on weapon usage by period have been added. The basic idea of SELECTOR/HEAVY ATTACK is to buy whatever weapons are required to fight a cost-effective war, so it would seem illogical to include constraints on the usage of a particular weapon. The trouble is that certain weapons (AGM-65A/B Maverick air-to-ground missiles, for example) are no longer in production but still quite effective. The only realistic way to handle such weapons is to constrain their usage to the size of the current stockpile.

Note that the current system includes no budget constraints, even though it is a principally budgetary tool. SELECTOR utilizes cost inputs in determining the most cost-effective tactic, but weapon usage is not actually constrained by any budget. The HEAVY ATTACK output can therefore be interpreted as the classic military "requirements" for weapons with some implied budget level B. The idea that B should be an input, rather than an output, requires a fundamentally different view of the problem.

We continue to pursue enhancements and reformulations. Boger and Washburn [1985] describe an alternate nonlinear objective function where the parameter c_j has a physical interpretation. They also describe how the organization of computations in SELECTOR could lead to overutilization of weather-specialized weapons and weather robust aircraft. Wirths [1989] develops several prototypic reformulations using the GAMS/ MINOS system (e.g., Murtagh and Saunders [1984], Brooke, et al. [1992]). Amongst other things he derives a differential equation for which (3) is a solution, shows that the impact of using a linear objective function is not as great as had previously been thought, and asserts that the myopic approach to optimization over time is possibly of more concern. Utilizing a linear objective function would of course open up a great many other possibilities for reformulation, including the adoption of the conceptually simpler, but larger TAM (Might [1987]), which includes subscripts for weapons, weather, and, according to Jackson [1988], one-dimensional sortie range and time period too. However, TAM utilizes budget constraints, and our computational experience with TAM, as well as that of Jackson [1988], shows that it is very time consuming to solve in spite of its linear objective function.

CONCLUSION

We chose HEAVY ATTACK for this expository paper for some reasons not yet discussed. HEAVY ATTACK has for some time been a favorite classroom example at the Naval Postgraduate School. HEAVY ATTACK is simple to explain and understand without resorting to excessive mathematics, can be used in hands-on homework and modified for experiments by students, yet exhibits all the features, man-machine interaction, and richness a decision support system should have. The system has been in use for many years, and its remarkable longevity and direct influence on billion-dollar decisions automatically enhance student interest and warrant study of its design and application. Even issues of client and analyst psychology, the influence of politics on decision making, and techniques for preserving run-to-run solution persistence and comparison of optimized results can be highlighted. Being a nonlinear optimization model, it also provides rich collateral mathematical material such as characterization of concavity, numerical analysis, function approximation, aggregation, and proofs of convergence.

When the problem description for HEAVY ATTACK is given as a homework formulation exercise, students immediately construct detailed linear models: we call this approach the "$x_{subscript\text{-}foreverything}$" method. Asked to provide answers to the problem under time pressure, students quickly discover the large size and data appetite of their models, and devise reasonable aggregation strategies sometimes reminiscent of HEAVY ATTACK. Required to interpret their answers to this problem, students face many of the paradoxes inherent in modeling.

HEAVY ATTACK is an important member of a standard set of models we use to test new optimization techniques. The fact that we have always maintained nonlinear optimization capability in all our systems has derived in part from consideration of this application. We admit some professional satisfaction that HEAVY ATTACK has evolved, with many cohort models, from a daunting computational feat to a keystroke quick application, even for a microcomputer.

ACKNOWLEDGMENTS

The Air Force Office of Scientific Research and the Office of Naval Research have supported our basic mathematical research in optimization and encouraged its application. The Naval Postgraduate School promotes research on important problems even when there is no direct benefit for the Navy. Insight, Inc. of Alexandria, Virginia, has provided optimization software for HEAVY ATTACK. We thank the Air Force for the opportunity to publish this account in the unclassified literature.

REFERENCES

Bausch, D. and Brown, G. 1988, "A PC Environment for Large-Scale Programming," *OR/MS Today*, 15, 3 (June), pp. 20—25.

Blackett, P. 1962, *Studies of War*, Hill & Wang, New York.

Boger, D. and Washburn, A. 1985, "Notes from the Stockpile Seminar," *NPS55-85-014*, Naval Postgraduate School.

Bradley, G., Buckingham, L, Brown, G., and Wallace, D. 1992, "New Tools for Optimizing USAF Sortie Allocation Planning," ORSA/TIMS, Orlando, Florida (April 26).

Brooke, A., Kendrick, D., and Meeraus, A. 1992, *GAMS: A User's Guide, Release 2.25*, The Scientific Press, South San Francisco, California.

Brown, G. and Olson, M., 1994, "Dynamic Factorization in Large-Scale Optimization," *Mathematical Programming* (to appear) .

Charnes, A. and Cooper, W. 1958, "The Theory of Search: Optimum Distribution of Search Effort," *Management Science, 5*, pp. 44—50.

Clasen, R., Graves, G., and Lu, J. 1974, "Sortie Allocation by a Nonlinear Programming Model for Determining Munitions Mix," *RAND Corporation Report R-1~11-DDPAE*, March.

Crawford, G. 1989, "The Air Force's Munitions Requirements Process (The Nonnuclear Consumables Annual Analysis)", *RAND Corporation Report N-2821-P/L*, May 31.

Department of Defense 1993, "Program Objective Memorandum for FY 1994—1999," Washington, D.C.

INSIGHT, Inc. 1990, "XS(F) Mathematical Programming System," Copyright 1990, Alexandria, Virginia.

Jackson, J. 1988, "A Taxonomy of Advanced Linear Programming Techniques," Masters Thesis in Operations Research, Air Force Institute of Technology.

Joint Technical Coordinating Group/Munitions Effectiveness 1980, "Joint Munitions Effectiveness Manual, Air to Surface, Weapons Effectiveness, Selection, and Requirements (Basic JMEM A/S)," 10 December.

Lord, P. 1982, "An Examination of the United States Air Force Optimal Nonnuclear Munitions Procurement Model," Masters Thesis in Operations Research, Naval Postgraduate School.

Might, R. 1987, "Decision Support for Aircraft and Munitions Procurement," *Interfaces*, 17, 5, September, pp. 55—62.

Murtagh, B. and Saunders, M. 1984, "MINOS - Version 5.0," Department of Operations Research, Stanford University.

Silicon Valley Software 1991, "SVS C3 Code Construction Series," San Mateo, California.

Wallace, D. 1992, "Analysis Tools for United States Air Force Sortie Optimization and Munitions Planning," Masters Thesis in Operations Research, Naval Postgraduate School.

Washburn, A. 1989, "Finite Methods for a Nonlinear Allocation Problem," *NPS55-89-003*, Naval Postgraduate School.

Wirths, K. 1989, "A Nonlinear Programming Model for Optimized Sortie Allocation," Masters Thesis in Operations Research, Naval Postgraduate School.